

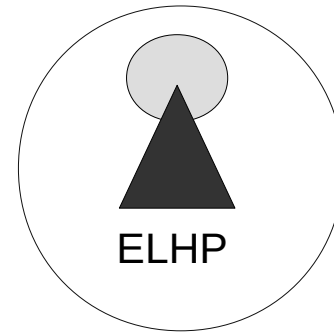
The conspiracy: What books say

- (nothing at all)
- ... but we won't say anything about that.
- ... but it's rather contrived so we won't say anything about that.
- ... but it doesn't turn up in actual applications so we won't say anything about that
- ... see this other book for information
- description but no example
- description and one example – always the same one

(Informal literature survey by Rachel Traylor (@mathpocalypse) and myself)

A Conspiracy of Silence, Vagueness and/or Dark Hinting

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ELHP



Contains TWO examples!

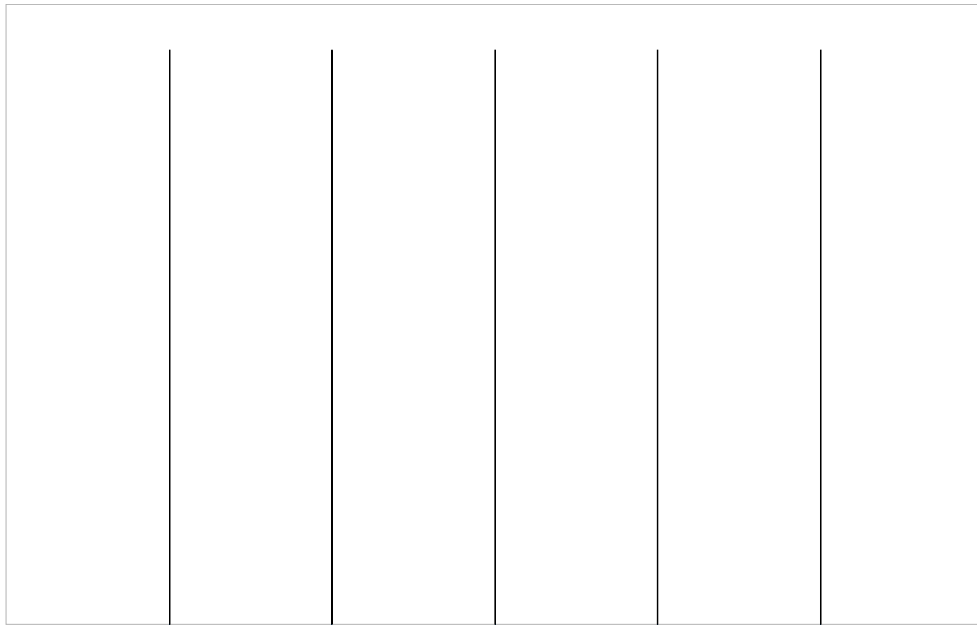
Discrete Distributions

An example

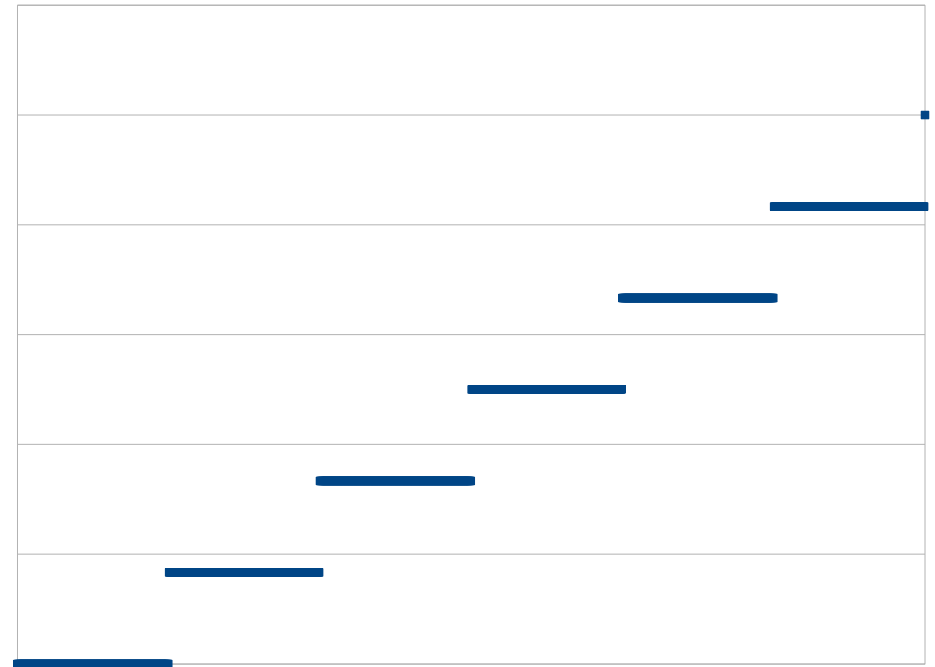
- Throw a fair six-sided die, divide result by 6.
 - Six possible values between 0 and 1, probability $1/6$ each.
 - Mass function
-
- Famous discrete distributions: Geometric, Poisson, Binomial.

Discrete: Mass function and CDF

Mass function



CDF



JUMPY BITS!

Continuous Distributions

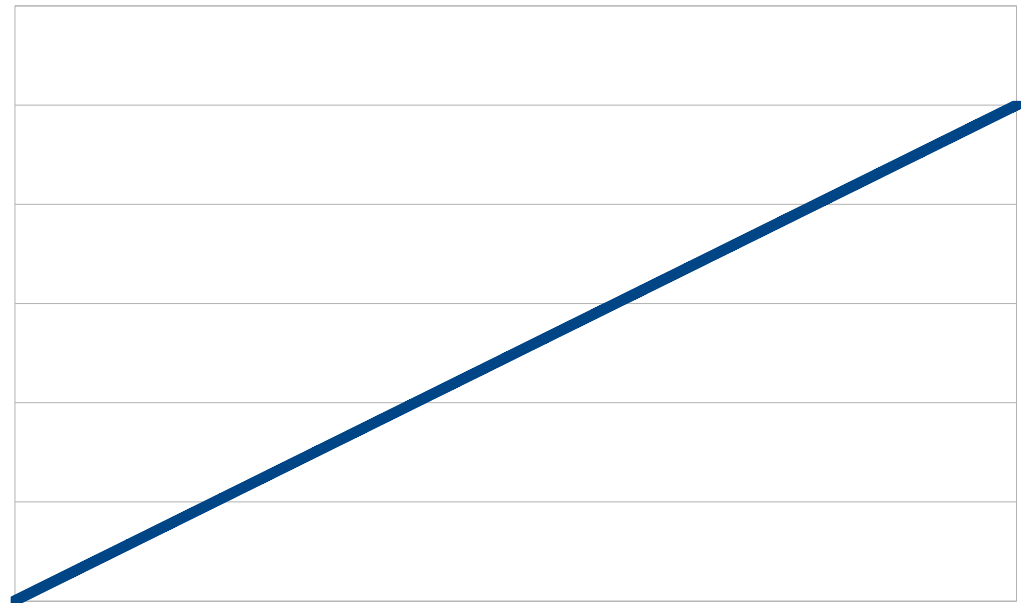
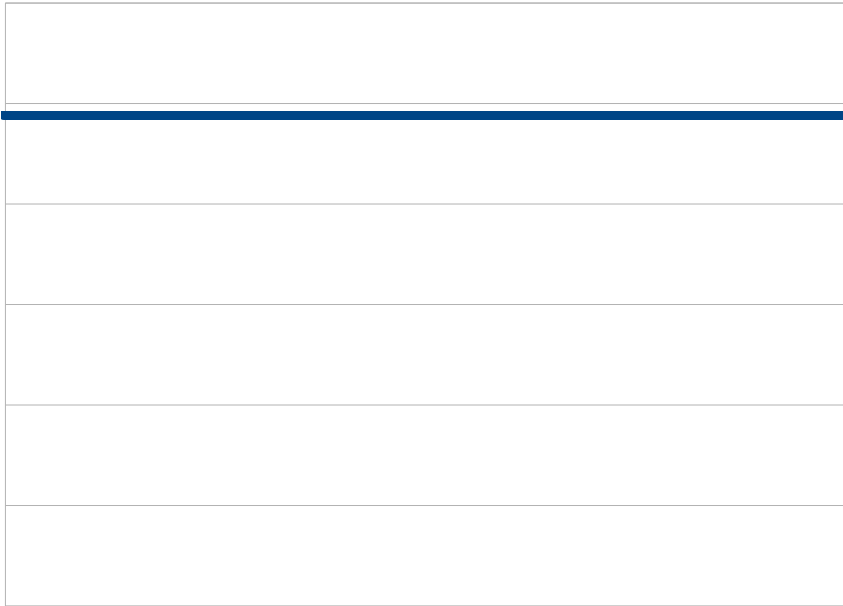
An example:

- Choose random real in $[0,1]$ uniformly.
 - All reals between 0 and 1 are possible.
 - Density function
-
- Famous continuous distributions: Normal (aka Gaussian), Exponential, Uniform (on some range).

Continuous: Density function and CDF

Density

CDF

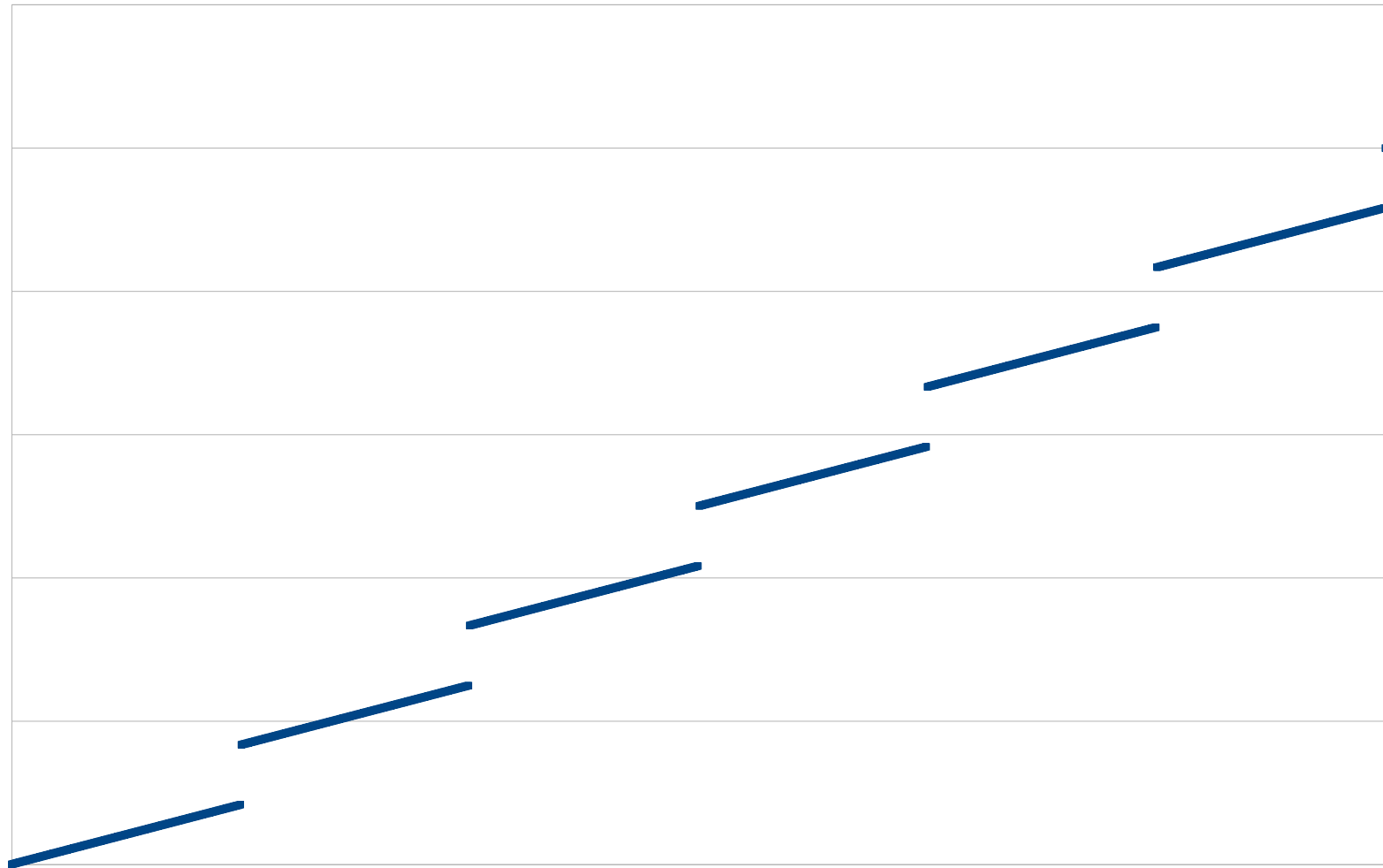


SLOPEY BITS!

Hybrid of previous two

- Throw a fair coin
- If heads, take value from discrete example
- If tails, take value from continuous example
- Our distribution is neither continuous nor discrete. Has neither mass nor density. Still has C.D.F.

CDF of hybrid example



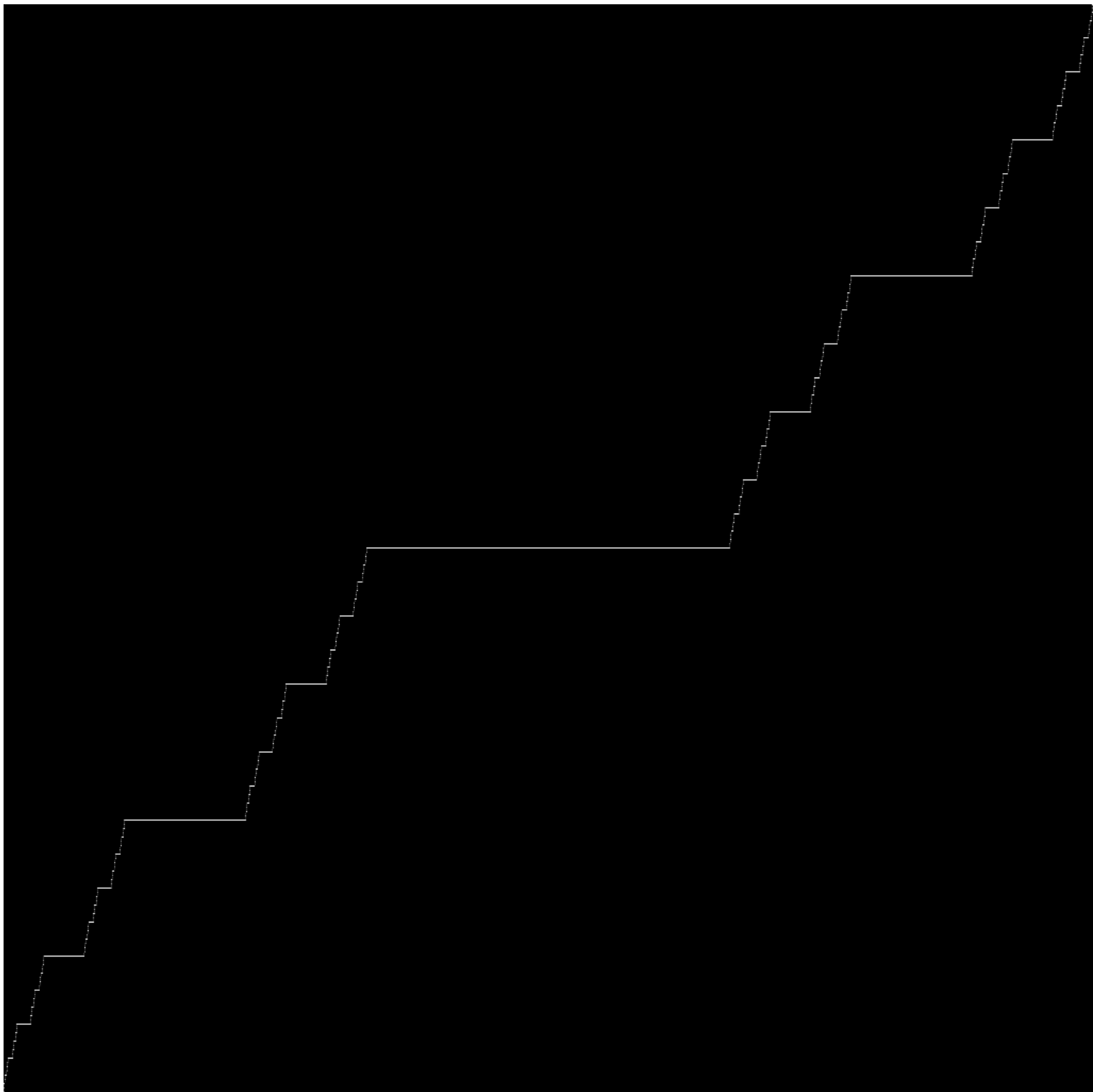
BOTH JUMPY AND SLOPEY BITS!

But that's not the conspiracy

- Just a mixture of two things we know. That's not new!
- There is a third basic kind of distribution about which many courses and books say little or nothing
- Instead of BOTH jumps and slopes, try NEITHER!
- These are called:
 - ~~Distributions from beyond space and time~~
 - ~~Red-headed step-distributions~~
 - ~~Probability distributions of the third kind~~
 - Singular distributions

And here is one!

- Throw a coin infinitely often. Once for each positive integer.
- For each positive integer n where you get heads, award yourself $\frac{2}{3^n}$ pounds.
- What is distribution of your possible earnings?
- No mass or density. But here's the CDF.

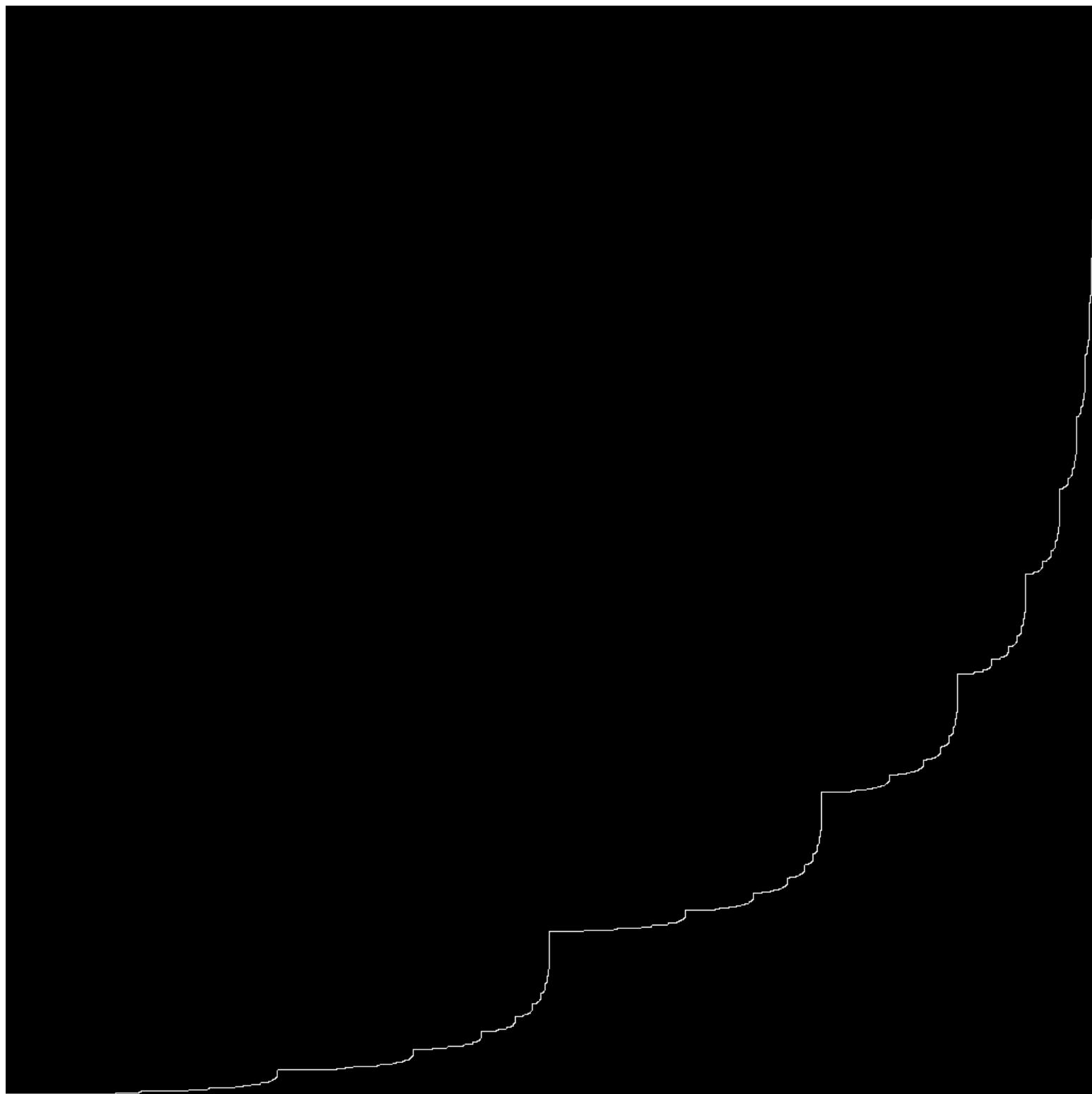


- CDF has derivative 0 almost everywhere but is continuous.
- If an example is given, it's always this one.
- Mean $1/2$, variance $1/8$
- Contrived? Hah! THIS IS MATHSJAM!

Video

Are they all basically like that?

- Lots of flat bits and some jiggery-pokery?
- No, they can be worse.
- e.g. you want contrived, try this one!



Where does this come from?

- Have x pounds (between 0 and 1)
- Want to reach 1 pound
- Can only bet on an unfair coin coming up heads with prob p ($0 < p < 1/2$).
- Always bet everything if $x < 1/2$ or just enough to reach 1 otherwise.
- $f(x)$ is probability of eventually reaching 1.

Where does this come from?

- $f(x)$ in context of problem not a CDF
- But satisfies requirements so use it as one!
- What are mean and variance?
- CDF is continuous, differentiable almost everywhere with derivative 0.
- This time, also strictly increasing!

Video

In general a probability distribution is a mix of three components: discrete, continuous and singular.

Ways to make more singular distributions:

- Fractal interpolation
- Stretching an existing one
- Copying and pasting bits of existing ones onto any distribution you like